SCAD ENGINEERING COLLEGE
Approved by AICTE & Affiliated to Anna University | ISO Certified Institution
Cheranmahadevi, Tirunelveli

DEPARTMENT OF MECHANICAL ENGINEERING

CE6306-STRENGTH OF MATERIALS

CE6306 STRENGTH OF MATERIALS

UNIT I STRESS, STRAIN AND DEFORMATION OF SOLIDS

Rigid bodies and deformable solids – Tension, Compression and Shear Stresses – Deformation of simple and compound bars – Thermal stresses – Elastic constants – Volumetric strains –Stresses on inclined planes – principal stresses and principal planes – Mohr’s circle of stress.

UNIT II TRANSVERSE LOADING ON BEAMS AND STRESSES IN BEAM

Beams – types transverse loading on beams – Shear force and bending moment in beams –Cantilevers – Simply supported beams and over – hanging beams. Theory of simple bending–bending stress distribution – Load carrying capacity – Proportioning of sections – Flitched beams –Shear stress distribution.

UNIT III TORSION

Torsion formulation stresses and deformation in circular and hollows shafts – Stepped shafts–Deflection in shafts fixed at the both ends – Stresses in helical springs – Deflection of helical springs,
carriage springs.

UNIT IV DEFLECTION OF BEAMS

Double Integration method – Macaulay’s method – Area moment method for computation of slopesand deflections in beams - Conjugate beam and strain energy – Maxwell’s reciprocal theorems.

UNIT V THIN CYLINDERS, SPHERES AND THICK CYLINDERS

Stresses in thin cylindrical shell due to internal pressure circumferential and longitudinal stresses anddeformation in thin and thick cylinders – spherical shells subjected to internal pressure –Deformation
in spherical shells – Lame’s theorem.

DEPARTMENT OF MECHANICAL ENGINEERING

UNIT 1

Stress, Strain, and change in length relationship

Stress is proportional to strain within its elastic limit. This law is known as Hooke's law. The material will not return to original shape if the applied stress is more than E.

$\sigma \propto \varepsilon$ Stress - σ Linear Strain – ε

Therefore, $\sigma = E\varepsilon$ Where E Modulus of Elasticity or Young's Modulus.

$$\sigma = \frac{P}{A}$$

P – Load

A- Area of the section where the load is applied.

Stresses are three types tensile, compressive, and shear stress. Moment and torsion will produced any of these stresses.

Strain is nothing but deformation (change in length, breadth, height, diameter, therefore area or volume) of the body or material due to load. Therefore strain is change in dimension to the original dimension. It may be length or volume.

$$\varepsilon = \frac{\delta_L}{L}$$

δ_L – Change in length

L – Original length

Therefore by substituting the value of σ and ε in the Hook's law. Change in length is

$$\delta_L = \frac{PL}{AE}$$

$$\delta_L = \frac{4PL}{\Pi E d_1 d_2}$$ **uniformly varying circular section**

$$\delta_L = \frac{PL}{Et(a-b)} \log_e(a/b)$$ **uniformly varying rectangular section a>b**

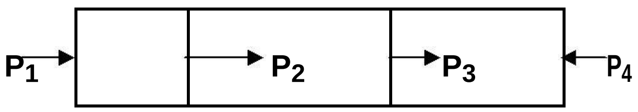
This is the fundamental equation to find change in length of any type of section or step section using principle of superposition method of varying load, length, area, and material. The change in length due to compressive load is taken as negative and positive for tensile load.

Types of problem

Both ends are free (to expand or shrink) determinate structure:

Total change in length is equal to algebraic sum of change in length of each section of its load P, length L, Area A, and Young's modulus E. These parameters may vary from section to section. The material is free to expand and shrink.

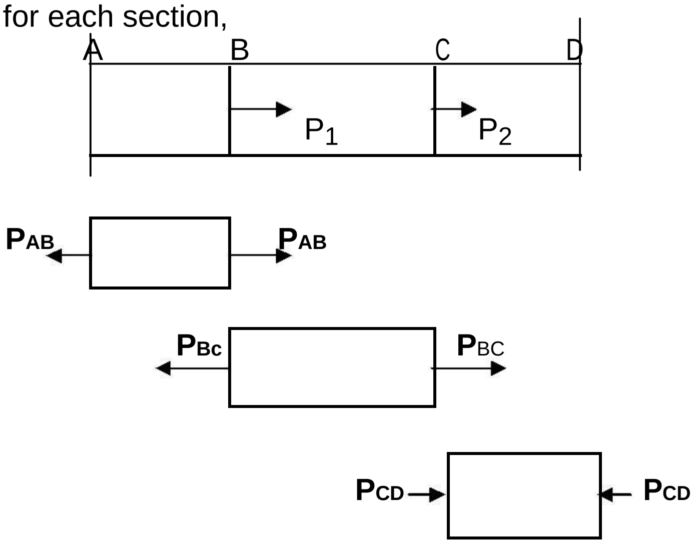
$$\delta_L = \delta_1 + \delta_2 + \delta_3 + \ldots + \delta_n$$



Both ends are fixed (cannot expand or shrink) indeterminate structure:

Total change in length is zero because the ends are fixed which will not allow the sections to expand or shrink. Load or stress is produced by expansion or shrinkage of the section is taken by the ends. Therefore ends carry some load or stress.

Using principle of superposition the reactions at the end of each section is found from free body diagram. Equate the direction of force in free body diagram to force applied for each section,



$$P_{AB} - P_{Bc} = P_1$$

$$P_{BC} + P_{CD} = P_2 \quad \textbf{Equations – (A)}$$

The equation shows that the section AB and BC is under tension and CD under compression. The direction of load in each section can be chosen as we desire, but if the final result is negative then the direction chosen is incorrect but the answer is correct. So in other words tensile force is actually a compressive force vice versa.

Sum of change in length of each section due to expansion is equal to sum of change in length of each section due to compression. The load P, length L, Area A, and Young's modulus E parameters may vary from section to section.

Expansion section = Compression section

$$\delta_1 + \delta_2 + ... + \delta_n = \delta_3 + \delta_4 + ... + \delta_n \quad \textbf{Equations – (B)}$$

Using equation A and B the problem can be solved.

Composite Material of Equal length

Reinforced Columns, Supporting load, Suspended load, Composite structure of equal length (example pipe inside a pipe) these problems can be solved with the following expression.

The change length is same for all materials in that structure. Example in reinforced concrete column (RCC), steel and concrete length change equally, similarly for supporting load, suspended load, and composite structure of equal length. Therefore to solve these problems use the following expressions.

Change in length of concrete = change in length of steel $\delta_{lc} = \delta_{ls}$ Equation – (A)

It is same as equation below for equal length only

$$\frac{\sigma_c}{E_c} = \frac{\sigma_s}{E_s}$$

For unequal length it is

$$\frac{\sigma_c L_c}{E_c} = \frac{\sigma_s L_s}{E_s}$$

The load P may be shared by two material equally or unequally.

$P = P_c + P_s$ P is Total load, P_c load taken by concrete and P_s steel. Or $P = A_c\,\sigma_c + A_s\,\sigma_s$ (B)

When the lengths of the composite material are equal by substituting B in A, find the stresses in the materials.

The ratio of Es/ Ec is known as modular ratio

Composite Material of Unequal length tubular section

1. Find the material or section whose length is shorter or longer than other material.
1. Calculate the load required to make the section of equal length using formula of δ_l.
2. This will give the remaining load that will be shared by both the sections.
3. At this point onwards it is similar to composite material of equal length.

Bolt and Nut:

Load in bolt = Load in tube

$$\sigma_b A_b = \sigma_t A_t$$

Change in length is sum of change in length in bolt and change in length in tube.

$$\delta = \delta_b + \delta_t$$

Thermal Stresses:

When there is increase in temperature the material expands this will produce stress. This is known as thermal stress.

$$\delta_l = L\,\alpha\,t$$

Thermal stresses when the material is not allowed to expand:

$$\varepsilon = \frac{\delta_l}{L} = \alpha\, t \text{ ---------Equation (A)}$$

$$\sigma = E\varepsilon \text{ ---------Equation (B)}$$

Substituting A in B

$$\sigma = E\,\alpha\, t$$

Thermal stresses when the material is allowed to expand to a length Δ:

$$\delta_l = l\,\alpha\, t -$$

$$\varepsilon = \frac{\delta_l}{L} = \frac{l\ \alpha t -}{l} \quad \text{Equation (C)}$$

Therefore stress is **$\sigma = E\varepsilon$.**

<u>Thermal Stresses in composite bars:</u>

Therefore load in brass is equal to load in steel because temperature is assumed to be uniform.

$$\sigma_s A_s = \sigma_b A_b \ \text{ - (A)}$$

Change lengths are therefore strains are equal thus,

$$\alpha_b t - \frac{\sigma_b}{E_b} = \alpha_s t + \frac{\sigma_s}{E_s} \text{ ----------Equation (B)}$$

Substituting equation A in B to find the stresses in the material.

When the thermal coefficient of one material is larger than the other then that material will be under compression and the other material will be under tension. Thus brass is under compression and steel is under tension in our example.

<u>Volumetric Strain:</u>

Change in volume to the original volume is known as volumetric strain.

Poisson ratio: It is the ratio of lateral strain to the linear strain. It is denoted by symbol **μ**

$$\mu = \frac{\text{lateral strain}}{\text{linear strain}} \text{ or } m = \frac{1}{\mu}$$

Change in volume due to axial load in all three directions for a cube or cuboids

$$\frac{\delta_v}{V} = \frac{1}{E}(\sigma_x + \sigma_y + \sigma_z)(1-2\mu)$$

This equation is valid only when all the loads are applied as tensile load. The same equation can be used for the following loads,

1. Compressive load change to minus sign to that direction only for the above formula.
2. Load only in one direction the remaining stresses are zero.
3. Load in two directions the remaining stress is zero.

Change in volume due to axial load for a cylindrical rod

Change in diameter in cylinder is $\boldsymbol{\varepsilon_c = \delta_d / d}$

Change in length in cylinder is $\boldsymbol{\varepsilon_l = \delta_l / l}$

Therefore change in volume of cylindrical rod;

$$\frac{\delta_v}{V} = \varepsilon_l - 2\varepsilon_c$$ (Minus sign lateral strain are compressive forces) **OR**

$$\frac{\delta_v}{V} = \frac{1}{E}(\sigma_x)(1-2\mu)$$ Where, σ_y and σ_z are zero because load in one direction only.

Three important moduli's are Elasticity, Bulk, and Rigidity

Modulus of Elasticity

$$E = \frac{\sigma}{\varepsilon} \qquad \varepsilon \text{ from } \delta_l = \frac{PL}{AE}$$

Bulk Modulus: Ratio of stress over volumetric strain

$$K = \frac{\sigma}{(\delta_v/V)}$$

It is also same as when related with E

$$K = \frac{mE}{3(m-2)}$$

Modulus of Rigidity: Shear stress is proportional to shear strain.

τ α φ τ

= Cφ

$$= \frac{mE\,C}{2(m + 1)}$$

Strain Energy:

Strain Energy in Gradual Load

U = Average Load x Change in length

= stress x strain x volume

Substituting the value of stress, strain, and volume of the section

$$U = \frac{P\,\delta_L}{2} \qquad \delta_L = \frac{PL}{AE}$$

The stress σ due to gradual load is P/A.

$$U = \frac{\sigma^2 V}{2E}$$ This is the strain energy stored in a body. **– Equation (A)**

Strain Energy in Sudden Load

The stress due to sudden load is found by equating the equation (A) in the following equation. (B)

$$U = P \times \delta_L$$ **---- Equation (B)**

$$\frac{\sigma^2 V}{2E} = P \times \delta_L$$

Therefore stress produced due to sudden load is

$$\sigma = \frac{2P}{A}$$

Strain energy due to sudden load is found by substituting the stress σ due to sudden load in the following equation

$$U = \frac{\sigma^2 V}{2E}$$

Strain Energy in Impact Load

U = Load x (height + Change in length)

The stress σ due to impact load when δ_L is negligible

$$\sigma = \frac{\sqrt{2EPh}}{AL}$$

The stress σ due to impact load when δ_L is not negligible

$$\sigma = \frac{P}{A}\quad 1 + \sqrt{1+2Eh(PL)}$$

Strain energy due to impact load is found by substituting the stress σ due to impact load in the following equation.

$$U = \frac{\sigma^2 V}{2E}$$

Principal Stress and Strains

Principal Plane: - It is a plane where shear force is zero is called principal plane.
Principal Stress: - The normal stress on the principal plane is called principal stress. **Obliquity**: - It is angle between the resultant stress and normal stress.
Mohr's circle: - It is a graphical (circle) method to find the stresses and strains on a plane.

Principal Plane and Stresses can be solved by

1. Analytical Method – Solving horizontal and vertical stresses to find the normal stress and shear stress using trigonometry method.
2. Graphical Method – Mohr's circle method

Analytical Method:

The equation is solved assuming σ_x and σ_y as tensile stresses as positive and τ_{xy} shear stress clockwise as positive to major principal stress. Simply change the sign if stresses are opposite.

General equation to find the normal stress:

$$\sigma_n = \sigma_x \sin^2\theta + \sigma_y \cos^2\theta - 2\tau_{xy}\sin\theta\cos\theta$$

General equation to find the shear stress:

$$\sigma_t = \sigma_x \sin\theta\cos\theta - \sigma_y \sin\theta\cos\theta + \tau_{xy}\sin^2\theta - \tau_{xy}\cos^2\theta$$

The resultant stress, $\sigma_R^2 = \sigma_n^2 + \sigma_t^2$

Maximum principal stress

$$\sigma_{max} = \frac{(\sigma_x + \sigma_y)}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2}$$

Minimum principal stress

$$\sigma_{min} = \frac{(\sigma_x + \sigma_y)}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2}$$

Maximum shear stress

$$\sigma_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2}$$

Graphical Method - Drawing Rules of Mohr's Circle:

1. Fix the origin (0,0) that is (x,y) at convenient place in the graph.
2. X – axis to locate axial stress for both x and y directions.
3. Y – axis to locate shear stress for clockwise and anti clockwise shear.
4. Tensile stress is positive along x axis right of origin.
5. Compressive stress is negative along x axis left of origin.
6. Clockwise Shear stress is positive along y axis upward of origin.
7. Anti clockwise shear stress is negative along y axis downward of origin..
8. When there is no shear force (τ_{xy}= 0) draw Mohr's circle from axial stresses. The centre of the Mohr's circle bisects axial stresses (σ_x,0) and (σ_y,0).
9. When there is shear force draw Mohr's circle from axial stresses and shear stress. The centre of the Mohr's circle bisects the line between (σ_x, τ_{xy}) and (σ_y, τ_{xy}).
10. Angle of inclination is to be drawn from point (σ_y, τ_{xy}) at centre of Mohr's to angle 2θ in clockwise direction.
11. Normal stress, and maximum and minimum principal stresses are taken from the origin along the x-axis of the Mohr's circle.
12. Maximum shear stress is the radius of the Mohr's circle, and shear stresses are taken along the y-axis of the Mohr's circle.
13. The angle between the resultant stress and normal stress in angle of oblique.

In the Mohr's circle shown below σ_x and σ_y are tensile. Shear stress λ_{xy} is clockwise to stress plane

σ_y.

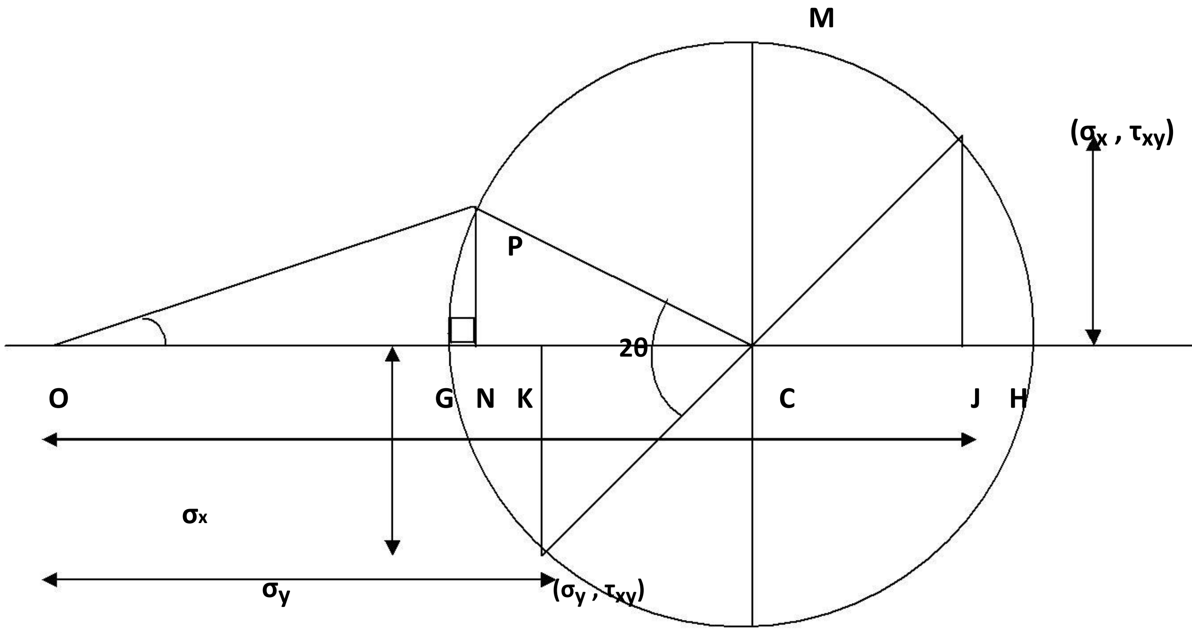
Observation from Mohr's circle: Given data:

σ_x = OJ is tensile stress on one plane. σ_y = OK is tensile stress on other plane. λ_{xy} is shear force on the pl

Result from Mohr's Circle:

CM = radius of Mohr's circle = λ_{max} It is the maximum shear stress. QP = σ_t Shear stress at the point.

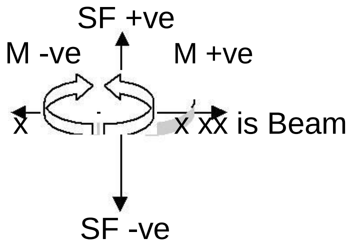
ON = σ_n Normal stress at the point. OP = σ_R Resultant stress at the point. OG = Minimum principal stre OH = Maximum principal stress. POG = angle of obliquity.



UNIT 2

Shear Force and Bending Moment

Sign rules followed for Shear Force and Moment from right side:



Any sign convention can be followed but it should be uniform throughout the problem. We have chosen upward load or shear force as positive and downward load or shear force as negative. Similarly take clockwise moment as negative and anticlockwise moment as positive.

Cantilever Beam:

1. Simply add the load from right to find the shear force at various points. Upward SF minus downward SF will give SF at a point it may be +ve or –ve SF.
2. Multiply the load with distance to find the moment at various points. Anti clockwise BM minus clockwise BM will BM at a point it may be +ve or –ve SF.
3. Shear force maximum at the support.
4. Moment maximum at the support and zero at free end.

Simply supported Beam:

1. Find the reactions at the supports.
2. When taking moment to find the reactions consider even the pure moment in the beam, be careful with the direction of the moment. Then follow the SF and BM diagram procedure to complete the figure.
3. Simply add the load from right to find the shear force at various points. Upward SF minus downward SF will give SF at a point it may be +ve or –ve SF.
4. Multiply the load with distance to find the moment at various points. Anti clockwise BM minus clockwise BM will BM at a point it may be +ve or –ve SF.
5. Moment is maximum where SF is zero for pure load only.
6. To find the maximum moment, find section where SF is zero equate upward load to downward load to distance x from a support. Take that distance to find the maximum moment.
7. Moments are zero at the supports.

.

Over hanging Beam:

1. Find the reactions at the supports.
2. When taking moment to find the reactions consider even the pure moment in the beam, be careful with the direction of the moment. Then follow the SF and BM diagram procedure to complete the figure.
3. Simply add the load from right to find the shear force at various points. Upward SF minus downward SF will give SF at a point it may be +ve or –ve SF.
4. Multiply the load with distance to find the moment at various points. Anti clockwise BM minus clockwise BM will BM at a point it may be +ve or –ve SF.

5. The moment changes the sign from positive to negative such point is known as point of contraflexure. To find the point of contraflexure find the section where MB is zero equate clockwise moments to anti clockwise moment to distance x from a support.

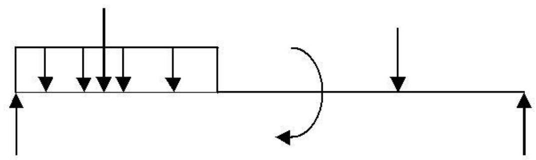
6. Moments are zero at the supports where there is no overhanging, and at the over hanging end.

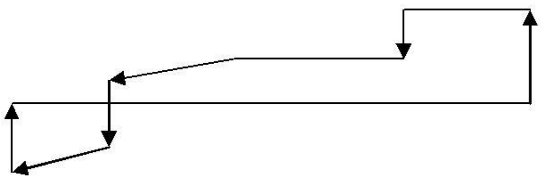
Drawing Shear force diagram:

1. Draw a reference line equal to length of the beam to scale.
2. Move the line up if SF is pointing upward or move the line down if SF is pointing downward.
3. When there is no load between loads draw horizontal line parallel to reference line.
4. Point load is represented by vertical line.
5. udl is represented by inclined line.
6. Uniformly varying load is represented by parabola line.
7. Ignore moment for shear force diagram.

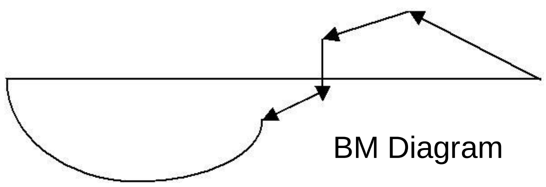
Drawing Bending Moment diagram:

1. Draw a reference line equal to length of the beam to scale.
2. Locate a point to find BM, clockwise is taken as negative and anti clockwise is taken as positive.
3. Draw an inclined line to the point if the moment is due to point load only between sections.
4. Draw a parabolic line to the point if the moment is due to udl load between sections.
5. Draw a vertical line for pure moment on the beam, downward if it is clockwise moment and upward if it is anti clockwise moment.





SF Diagram



BM Diagram

Bending Stress

M = WL/4 Simply support bean point load at mid span
$M = wL^2/8$ Simply support bean of udl throughout the span
M = WL Cantilever beam load at distance L from the support
$M = wL^2/2$ Cantilever beam of udl throughout the span

Stress is zero at centroid (NA) that is at distance y from the xx-axis and maximum at the top and bottom

We know,

$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$

M – Bending moment or Moment may vary depending on the load example **I –** Moment of Inertia.
σ – Stress due to bending moment. To find σ_c then $y = y_c$ and to find σ_t then $y = y_t$
y - Centroid of the section about xx axis (NA). To find σ_c then $y = y_c$ and to find σ_t then $y = y_t$ **E –** Modulus of Elasticity or Young's modulus.
R- Radius of curvature due to bending.

For symmetric section value of $\sigma_c = \sigma_t$ because $y_c = y_t$ example, rectangle, circular, and symmetric I section. That is N.A will be at mid point.

The value y_c = y from the bottom to NA for beam under compression and y_t = y from the top to NA for beam under tension. To find the safe Load or moment find the value of **σ_c/y_c and σ_t/y_t** and take the least value for safe design.

$I = bd^3/12$ Rectangular section and y = d/2
$I = \pi(D_o – D_i)/64$ for hollow pipe and solid rod y = Do/2 for solid pipe $D_i = 0$

Centroid (NA) of total section y = sum of (area of each section x centroid of each section from xx axis) divided by sum of (area of each section) Ref: figure

$$y = \frac{a_1y_1 + a_2y_2 + \ldots a_ny_n}{a_1 + a_2 + \ldots a_n}$$

Substitute the value y in the moment of inertia equation.

$$I = \frac{b_1d_1^3}{12} + a_1(y_1 - y)^2 + \frac{b_2d_2^3}{12} + a_2(y_2 - y)^2 + \ldots \frac{b_nd_n^3}{12} + a_n(y_n - y)^2$$

Shear Stress

Stress is caused due to Shear force or load. The shear load is right angle to the section. Shear Stress is zero at the top and bottom of the section and it is the maximum at centroid (NA) distance y from the xx-axis.

$$\tau = \frac{FAy}{Ib}$$

τ - Shear stress at a point F – Shear load A – Area of the section considered.
y – Centroid distance of the section considered from the Neutral axis of the whole section. I – Inertia of the whole section b – Width of the section considered.

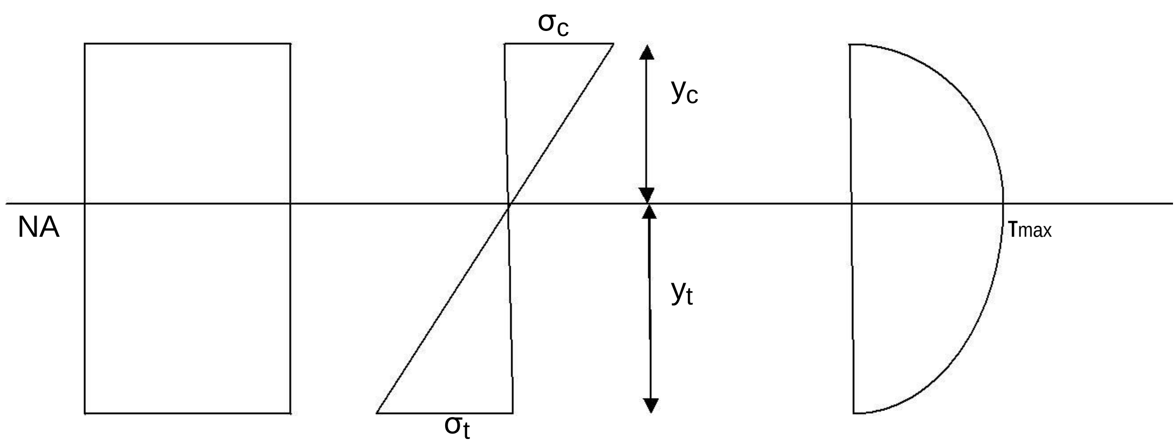
Shear stress in
flange y = D/2
+ d_1 A = B x D
b = B

Shear stress in beginning of
web y = D/2 + d_1
A = B x D
b = b

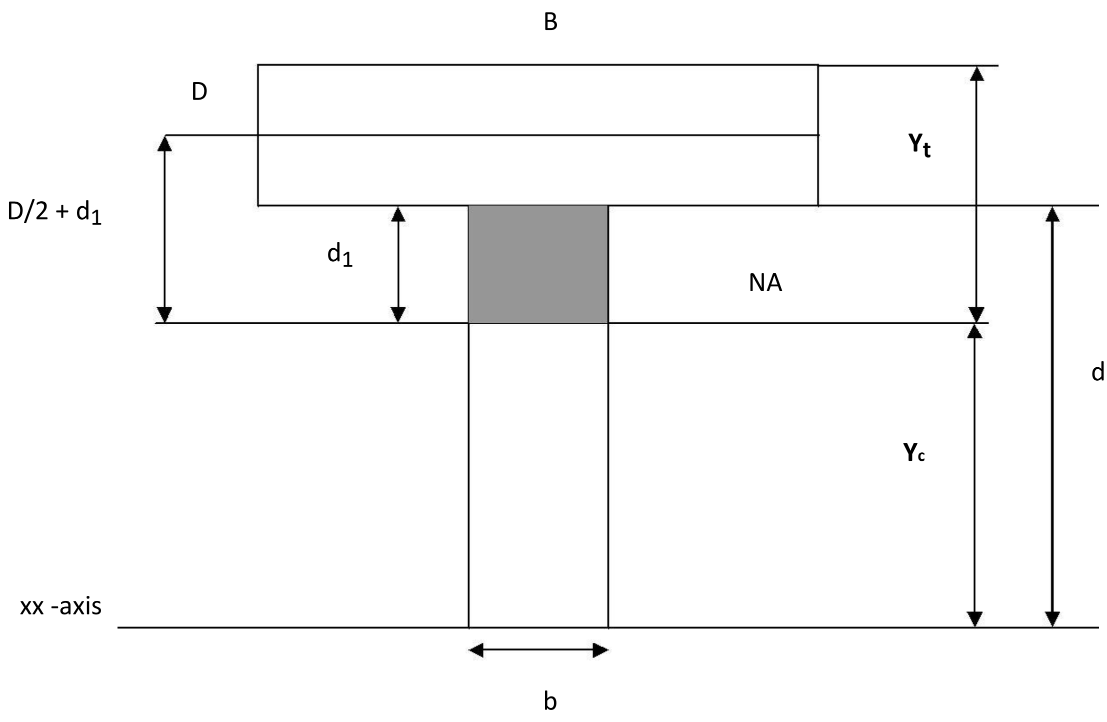
Shear stress in neutral axis (Maximum)

Ay = (B x D) x (D/2 + d_1)+ (b x d_1) x d_1/2 and b = b

d_1 – can be found only after finding neutral axis NA (Y).



Bending Stress Diagram Shear Stress Diagram


B
D
Y_t
D/2 + d_1
d_1
NA
d
Y_c
xx -axis
b

UNIT 3

Torsion in Shaft

Simple or Single shaft

$$\frac{T}{I_p} = \frac{\sigma}{r} = \frac{G\theta}{L}$$ **is general equation to stress and twist due to torsion.**

T = Torque or Torsion or Angular Velocity obtained from power Ip = Polar moment of inertia is sum of I_{xx} and I_{yy}
σ = Shear stress in shaft r = radius of shaft
L = Length of shaft
θ = Angle of twist in radian.
G or C = Modulus of rigidity

Convert to radian $180^0 = \Pi$ radian.

$$P = \frac{2\Pi N T_{mean}}{60}$$

N - rpm
When P is in watt T will be N-m
and when P is in Kw T will be KN-m

Hollow shaft

$$I_p = \frac{\Pi (D^4 - d^4)}{32}$$

Ip = I/2 only for circular section

D - External dia and d – internal dia

Solid shaft d = 0
Therefore,

$$I_p = \frac{\Pi D^4}{32}$$

Ip = I/2 only for circular section

Strength of shaft is,

$$T = \frac{\Pi \sigma D^3}{16}$$

Angle of twist is,

$$\theta = \frac{32\, TL}{G \Pi D^4}$$

Torsional rigidity is the product of G and Ip which is GIp. Zp is known as polar modulus which is ratio of Polar inertia over the distance from NA.

Note: To design the safe diameter, find the diameter required for stress τ and diameter required for twist θ and select which ever is larger.

Shafts in series:

Conditions: Torque is same in shafts $T_1 = T_2$

Twist $\theta = \theta_1 + \theta_2$ Shafts rotate in same direction

Twist $\theta = \theta_1 - \theta_2$ Shafts rotate in opposite direction

Choose the least Torque between shafts for safe stress and angle of twist.

Shafts in parallel:

Conditions: Total Torque $T = T_1 + T_2$

Twist is same in both shaft $\theta_1 = \theta_2$

The shafts may be of same material or different material, which is known as composite shaft.

Strain energy or Torsional resilience in shaft:

It is the amount of energy stored when the shaft is in twisted position.

Torsional energy U = Average Torque x angle of twist

$$U = \frac{T \times \theta}{2} \quad \text{--------(A)}$$

$$\theta = \frac{\sigma L}{Gr}$$ substitute θ equation in (A)

and

$$T = \frac{\sigma I_p}{r}$$ substitute T equation in (A)

Therefore,

$$U = \frac{\sigma^2 V}{4G}$$

When U is divided by the volume of the shaft, is known as strain energy per unit volume.

Shaft coupled:

The shaft is joined together when the length is not sufficient this is known as coupling of shaft. It is done in two methods.

1. Using bolts
2. Using key

Bolt method

T can be obtained from shaft expression for bolt and keyed shaft.

$$T = \frac{\sigma I_p}{r}$$ or from Power expression $$P = \frac{2\pi NT}{60}$$

T is torque in shaft which is transmitted to the coupled shaft through bolts or key. Therefore torque in bolts or key is equal to torque in shaft.

T = no. of bolts **x** area of bolt **x** stress in bolt **x** radius of bolt circle

Therefore $T = n \times \Pi d_b^2 \times \sigma_b \times D_b/2$

T – Torque in bolts
d_b- Diameter of
bolt σ_b – Stress in
bolt n – no. of bolts
D_b – Diameter of bolt circle
Bolt circle of Bolt pitch circle (D_b) is diameter of bolt circle.

Key method

T = area of key **x** stress in key **x** radius of shaft

Therefore $T = l_k \times b_k \times \sigma_k \times r$

T – Torque in Key
l_k- - Length of key
b_k – Breadth or width of
key σ_k – Stress in key
r – radius of shaft

Torsion in Springs

Classification of spring:

1. Moment
   1. Semi elliptical Leaf Spring – Like simply supported beam load at mid point.
   2. Quarter elliptical Leaf Spring – Like cantilever beam load at end point.
2. Torsion
   1. Helical Spring
      1. Closed coiled - Angle of helix α is less than 10^o
      2. Open coiled - Angle of helix α is more than 10^o

Closed Coiled Spring:

Moment is very less compared to torsion therefore moment is ignored.

$$\frac{T}{Ip} = \frac{\sigma}{r} = \frac{G\theta}{L}$$

P or W - Load to spring
R – Mean radius of coil
T - Torque or Torsion T = P x
R n – Number of coils or turns
L – Length of spring L = 2ΠRn
σ - Shear stress in spring
C or G - Modulus of rigidity
θ - Angle of twist in radian
Ip = Polar moment of inertia is sum of Ixx and
Iyy d - Diameter of spring
Spring index = D/d
Solid length = nd
Stiffness k = P/δ
Deflection δ = Rθ

$$I_p = \frac{\Pi d^4}{32} \quad \text{and} \quad \sigma = \frac{Tr}{I_p} \quad \text{and} \quad \theta = \frac{TL}{G\, I_p}$$

Therefore,

$$\sigma = \frac{16PR}{\pi d^3} \qquad \delta = \frac{64PR^3 n}{d^4 G}$$

WAHL'S FACTOR :

In order to take into account the effect of direct shear and change in coil curvature a stress factor is defined, which is known as Wahl's factor

K = Wahl' s factor and is defined as

$$K = \frac{4c-1}{4c-4} + \frac{0.615}{c}$$ Where C = spring index

= D/d

if we take into account the Wahl's factor than the formula for the shear stress becomes $\tau_{max^m} = \frac{16.T.k}{\pi d^3}$

Spring frequency of vibration:

$$f = \frac{\sqrt{g/\delta}}{2\Pi}$$ unit Hz (Hertz)

Springs in series:

Conditions: Load is same in springs $P = P_1 = P_2$

Total Change in length or deflection $\delta = \delta_1 + \delta_2$

Therefore equivalent stiffness is

$$\frac{1}{k_e} = \frac{1}{k_1} + \frac{1}{k_2}$$

Springs in parallel:

Conditions: Load is shared by springs $P = P_1 + P_2$

Total Change in length or deflection is same $\delta = \delta_1 = \delta_2$

Therefore equivalent stiffness

is $k_e = k_1 + k_2$

Strain energy in spring:

It is the amount of energy stored when the spring is in twisted position. Strain energy U = Average Torque x angle of twist

$$U = \frac{T\theta}{2} \quad \text{or} \quad \frac{P\delta}{2}$$

Therefore

$$U = \frac{\sigma^2 V}{4G}$$

When U is divided by the volume of spring, is known as strain energy per unit volume.

Open Coiled Spring Axial deflection:

It is same as closed but the angle helix should be considered to solve the problem. The angle of helix should be greater than 10 degree. It generates torsion as well as moment

Torsion T = PRcosα   Moment M = PRsinα   Alpha α is angle of helix

$$L = \frac{\Pi Rn}{\cos\alpha} = 2\,\Pi\,Rn\sec\alpha$$

Angle of twist due to torsion, by substituting L and T in the equation below

$$\frac{T}{I_p} = \frac{G\theta}{L}$$

We get,

$$\theta = \frac{64PR^2 n\cos\alpha\sec\alpha}{Gd^4}$$ This is the angle of twist due to torsion

Angle of deflection due to moment by substituting the value of M, L and φ in the equation below

$$\frac{M}{I} = \frac{E\varphi}{L}$$

We know, Moment M = PRsinα

φ = ML/EI – Slope equation for the moment.

We get,

$$\varphi = \frac{128\,PR^2\,n\sin\alpha\sec\alpha}{E\,d^4}$$ This is the angle due to bending moment

Work done is due to torsion and moment for open coiled spring. Work done or strain energy U = Average load x distance = P δ /2

$$U = \frac{T\theta}{2} + \frac{M\varphi}{2}$$

$$\frac{P\delta}{2} = \frac{T\theta}{2} + \frac{M\varphi}{2}$$

Substituting the value of θ and φ δ is

$$\delta = \frac{64PR^3 n \sec\alpha}{d^4}\left[\frac{\cos^2\alpha}{G} + \frac{2\sin^2\alpha}{E}\right]$$

Therefore stress in coil is due to torsion and moment therefore,

$$\sigma_t = \frac{16PR\cos\alpha}{\pi d^3} \text{ and } \sigma_m = \frac{32PR\sin\alpha}{\pi d^3}$$

Total stress in the coil is $\sigma = \sigma_t + \sigma_m$

When, α = 0 it is same as closed coil spring.

Strain energy of open coiled spring is

$$U = \frac{P\delta}{2}$$

Angular rotation of open coiled spring:

β = θsinα – φcosα (Substituting the value of θ and φ)

$$\beta = \frac{64PR^2 n \sin\alpha}{d^4}\left[\frac{1}{G} - \frac{2}{E}\right]$$

E – Modulus of elasticity

Leaf Spring:

The reaction is like simply supported beam with concentrated load at mid span. Leaf springs do not develop any torsional stress. General equation is

$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$

Derivation of Semi elliptical Leaf Spring:

$$M = \frac{PL}{4}$$ **P – load and L – Span of Spring**

Since leaf spring consist of layer of plates of thickness t and breadth b. Therefore I for a plate is

$$I = \frac{nbt^3}{12}$$ **therefore,**

n – Number of leaves or plates

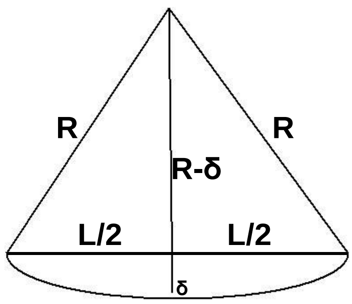
I for n plates is I x n

y = t/2 N.A of plate

We know to find stress,

$$\frac{M}{I} = \frac{\sigma}{y}$$

$$\sigma = \frac{3PL}{2nbt^2}$$ **equation (A)**



Deflection equation: **Deflection δ from pythagros theorem.**

$$\delta = \frac{L^2}{8R}$$ **equation (B)** ignoring small values.

Where, L is Length of plate and R radius of curvature due to bending of spring.

Substituting B in the following equation

$$\frac{\sigma}{y} = \frac{E}{R}$$

$$\delta = \frac{\sigma L^2}{4Et}$$ **substitute for σ from equation (A)**

$$\delta = \frac{3PL^3}{8nEbt^3} \quad \text{or} \quad P = \frac{8nEbt^3\delta}{3L^3}$$

Stiffness

$$= \frac{P\ k}{\Delta}$$

[Derivation of Quarter elliptical Leaf Spring:]

M = PL **P – load and L – Span of Spring**

Since leaf spring consist of layer of plates of thickness t and breadth b.
Therefore I for n plates is

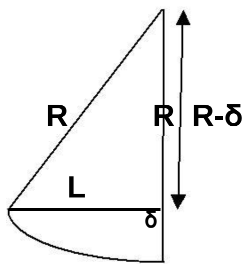
$$I = \frac{nbt^3}{12} \quad \textbf{therefore,}$$

n – Number of leaves or plates
I for n plates is I x n
y = t/2 N.A of plate

We know to find stress,

$$\frac{M}{I} = \frac{\sigma}{y}$$

$$\sigma = \frac{6PL}{nbt^2} \text{ equation (A)}$$



Deflection δ from pythagros theorem.

Deflection equation:

$$\delta = \frac{L^2}{2R} \quad \textbf{equation (B)} \text{ ignoring small values.}$$

Where, L is Length of plate and R radius of curvature due to bending of spring.

Substituting B in the following equation

$$\frac{\sigma}{y} = \frac{E}{R}$$

$$\delta = \frac{\sigma L^2}{Et} \text{ substitute for } \sigma \text{ from equation (A)}$$

$$\delta = \frac{6PL^3}{nEbt^3} \quad \text{or} \quad P = \frac{nEbt^3\delta}{6L^3}$$

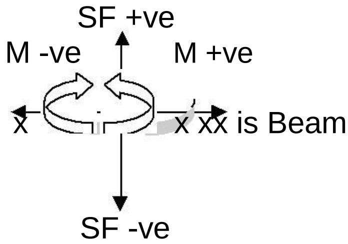
Stiffness

$$k = \frac{P}{\delta}$$

UNIT 4

Deflection of Beams

Sign rules followed for Shear Force and Moment from right side:

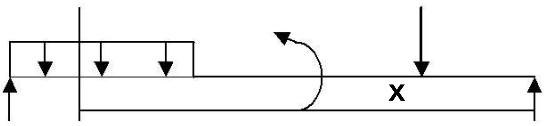


Note: Convert all loads to KN and length to Metre. Substitute the value of EI at the end. These will minimize the error during calculation. It is same as SF and BM in unit II. But only general BM equation is found at distance x by considering all loads and moments and equating it to the general equation EI $d^2y/dx^2 = Mx$.

Double Integration or Macaulay Method:

Cantilever:

1. Take moment Mx. for a distance x from free end.
2. While taking the moment all loads should be considered.
3. The concentrated moment should be taken as it is. Ex $150(x-5)^{0.}$ Clockwise or anti clockwise watch the moment direction.
4. The point load moment is, load x distance. Ex 20(x-1) clock wise.
5. The udl load moment for top load is, load x distance. Ex 10(x)(x)/2. This is for load distributed through out the span. Some times it may be required to counter the downward load. It is clock wise moment.
6. The udl load moment for counter load is load x distance. Ex 10(x-2)(x-2)/2. This is for load distributed from x-2 distance from one end to the other end of the support. Some times it may be required to counter the upward load. It is anti clock wise moment.
7. Clockwise is negative (-) and anti-clockwise is positive (+).
8. Equate the moment to the general equation EI $d^2y/dx^2 = Mx$.
9. Integrate once the equation will be slope equation.
10. Integrate once again the equation will be deflection.
11. Constants are found by boundary conditions, slope dy/dx and deflection y is zero at support for cantilever.
12. Substitute the constant to the slope equation that would be final equation of the slope. It is used to find slope anywhere along the length of beam.
13. Substitute the constant to the deflection equation that would be final equation of the deflection. It is used to find deflection anywhere along the length of beam.
14. While substituting the value of x to the equations ignore the negative number section of the equation.



Simply Supported Beam or Overhanging beam:

1. Find reaction at the support
2. Take moment Mx. for a distance x from any right support.
3. The concentrated moment should be taken as it is. Ex $150(x-5)^{0.}$ Clockwise or anti clockwise watch the moment direction.
4. The point load moment is, load x distance. Ex 20(x-1) clock wise.
5. The udl load moment for top load is, load x distance. Ex 10(x)(x)/2. This is for load distributed through out the span. Some times it may be required to counter the downward load. It is clock wise moment.
6. The udl load moment for counter load is load x distance. Ex 10(x-2)(x-2)/2. This is for load distributed from x-2 distance from one support to the other end of the support. Some times it may be required to counter the upward load. It is anti clock wise moment.
7. Clockwise is negative (-) and anti-clockwise is positive(+).
8. Equate the moment to the general equation $EI\, d^2y/ dx^2 = Mx$.
9. Integrate once the equation will be slope equation.
10. Integrate once again the equation will be deflection.
11. Constants are found by boundary conditions deflection y is zero at supports.
12. Substitute the constant to the slope equation that would be final equation of the slope. It is used to find slope anywhere along the length of beam.
13. Substitute the constant to the deflection equation that would be final equation of the deflection. It is used to find deflection anywhere along the length of beam.
14. While substituting value of x to the equations ignore the negative number section of the equation.

Moment Area method Cantilever:

1. Draw Bending moment Diagram.
2. The total area of the BM diagram will give the slope at free end.
3. To find the slope at the other point in the beam. Find the area of the BM diagram from the support to that point, that area would give the slope at that point. A/EI
4. The total area of the BM diagram multiplied by centroid from free end will give the deflection at the free end. Ax/EI x is centroid from point of deflection to be found.
5. To find the deflection at the other point in the beam. Find the area of the BM diagram from the support to that point multiplied by centroid from that point. That would give the deflection at that point.

Simply Supported Beam Point load at mid point and/or udl through out the span:

1. Find reaction at the supports.
2. Draw Bending moment Diagram.
3. The total area of the BM diagram divide by two will give the slope at each support.
4. To find the slope at the any point in the beam. Slope at a support minus area of the BM diagram from the support to that point, that area would give the slope at that point. Slope = Area of BM/EI.
5. To find the deflection at any point in the beam. Find the area of the BM diagram from the support to that point multiplied by centroid from that point. That would give the deflection at that point.

Conjugate method Cantilever:

It is a modification of Moment Area Method. It is effective where the inertia of section is different along the length of the beam. Conjugate method for cantilever is almost same as moment area method of cantilever.

1. Draw Bending moment Diagram of the given load.
2. The total area of the BM diagram will give the slope at free end.
3. The sum of the area of the BM diagram at varying inertia from a point to the support would give the slope at that point. $\Sigma A_n/EI$
4. The sum of moment of the BM diagram at varying section taken from a point to the support would give the deflection at that point. $\Sigma A_n X_n/EI$
5. To find the deflection at the other point in the beam. Find the area of the BM diagram from the support to that point multiplied by centroid from that point. That would give the deflection at that point.

Conjugate method simply supported beam:

1. Find the reaction of the given load and draw Bending moment Diagram.
2. Find the reaction of the support assuming the bending moment diagram as the load for varying inertia. This beam is known as conjugate beam.
3. The reaction at the supports will give the slope at the supports.
4. The upward load minus downward load of the conjugate beam will give slope at a point.
5. The moment taken at a point from the conjugate beam will give the deflection.

UNIT 5

Thin Cylinder

A cylinder whose thickness is 20 times less than the diameter is known as thin cylinder. When a liquid or gas flows through a pipe it cause stress to the pipe. There are three types of stresses induced.

Circumferential stress which will cause stress along the length of the cylinder. This will split the cylinder along the length. This is known as **Hoop stress**.

Longitudinal stress which will cause stress along the diameter of the cylinder. This will split the cylinder into two pieces along the diameter.

Radial stress is negligible so it is ignored.

Both the circumferential and longitudinal stresses are tensile. This is an important point to be remembered to find the change in volume.

Circumferential stress:

Resisting force by cylinder along length of pipe = force due to fluid pressure along length of pipe.

$$\sigma_c \, 2tL = pdL \quad \text{Thus } \sigma_c = \frac{pd}{2t}$$

Longitudinal stress:

Resisting force by cylinder along diameter of pipe = force due to fluid pressure along diameter of pipe.

$$\sigma_L \, \Pi \, Dt = p\,(\Pi \, D^2/4) \qquad \text{Thus } \sigma_L = \frac{pD}{4t}$$

Thus Hoop stress is **two times greater** than longitudinal stress. To find the thickness Hoop stress should be used.

Some times cylinders will have joints this will reduce the strength at the joint. Therefore and efficiency of joint (η) is included in the above expressions.

$$\sigma_c = \frac{pD}{2t\eta_c} \quad \text{and} \quad \sigma_l = \frac{pD}{4t\eta_L}$$

p – Pressure due to liquid or gas D – Diameter of cylinder
t – Thickness of cylinder
L – Length of cylinder
η – Efficiency of joint it may vary due to stress.

From Hooke's law strain $\varepsilon = \sigma/E$. This is the general equation for strain from which change in dimension can be found.

Strain due to circumferential stress cause increase in diameter

$$\varepsilon_c = \frac{pD}{2tE}\left(1 - \frac{1}{2m}\right)$$

Strain due to longitudinal stress cause increase in length

$$\varepsilon_l = \frac{pD}{2tE}\left(\frac{1}{2} - \frac{1}{m}\right)$$

Change in diameter in cylinder is $\boldsymbol{\delta_d = \varepsilon_c D}$

Change in length in cylinder is $\boldsymbol{\delta_l = \varepsilon_l L}$

Therefore change in volume of Cylinder:

$$\frac{\delta_V}{V} = \varepsilon_l + 2\varepsilon_c \text{ (OR) } = \frac{pD}{2tE}\left(\frac{5}{2} - \frac{2}{m}\right)$$

Thin Spherical Shells:

$$\sigma \Pi Dt = p(\Pi D^2/4)$$

$$\text{Thus } \sigma = \frac{pD}{4t}$$

$$\sigma = \frac{pd}{4t\eta}$$ **Stress due efficiency in joints**

Strain due to stress

$$\varepsilon = \frac{pD}{4tE}\left(1 - \frac{1}{m}\right)$$

Therefore change in diameter in cylinder is $\delta_d = \varepsilon d$

Change in volume of Cylinder:

$$\frac{\delta_v}{V} = 3\varepsilon$$

Thin cylinder wound to a unit Length:

Stress in cylinder due to initial winding = Stress in wire due to initial winding

$$\sigma_{c1}\, 2tL = \frac{\sigma_{w1}\, 2\pi\, d_w^2}{4} \cdot \frac{L}{D_w}$$ (2 because of two sides n=L/d_w)

$$\text{Thus} \quad \sigma_{c1} = \frac{\pi\, d_w\, \sigma_{w1}}{4t}$$

Stress due to internal pressure p = Stress in cylinder + Stress in wire

$$pDL = \sigma_{c2}\, 2tL + \sigma_{w2}\, \frac{2\pi\, d_w^2}{4} \cdot \frac{L}{d_w}$$

Longitudinal stress due to internal pressure p

$$\sigma_L = \frac{pD}{4t}$$

Change in Strain in wire = Final strain in wire – Initial strain in wire

$$\varepsilon_w = \frac{\sigma_{w2}}{E_w} - \frac{\sigma_{w1}}{E_w}$$

Change in Strain in cylinder = Final strain in cylinder – Initial strain in cylinder

$$\varepsilon_c = \frac{\sigma_{c2}}{E_c} - \frac{\sigma_l}{mE_c} - \frac{\sigma_{c1}}{E_c}$$

Change in Strain in wire = Change in Strain in cylinder that is $\varepsilon_w = \varepsilon_c$

$$\frac{1}{E_w}(\sigma_{w2} - \sigma_{w1}) = \frac{1}{E_c}(\sigma_{c2} - \sigma_{c1} - \sigma_L/m)$$

Since the initial strain in the wire σ_{w1}/E_w and the strain in cylinder σ_{c1}/E_c are the same, they are nullified, therefore the above equation is written as;

$$\frac{\sigma_{w2}}{E_w} = \frac{1(\sigma_{c2} - \sigma_L / m)}{E_c}$$

Final stress in wire $\sigma_w = \sigma_{w2} + \sigma_{w1}$ (Because both under tension)
Final stress in cylinder $\sigma_c = \sigma_{c2} - \sigma_{c1}$ (σ_{c2} is tensile σ_{c1} is compressive)