VALLIAMMAI ENGINEERING COLLEGE
DEPARTMENT OF MATHEMATICS
SUB CODE/ TITLE:MA6351- TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATION
QUESTION BANK
(COMMON TO ALL BRANCHES=THIRD SEMESTER)
UNIT-I (PARTIAL DIFFERENTIAL EQUATION)
PART-A

1. Form the partial differential equation by eliminating a, b from $z = (x^2 + a^2)(y^2 + b^2)$.

2. Form the PDE by eliminating the constants a and b form i)$z = ax^n + by^n$ **ii)**$z = ax^2 + by^2$

3. Find the partial differential equation of all planes passing through the origin.

4. Find the partial differential equation of the family of spheres having their centers on the line $x = y = z$.

5. Obtain partial differential equation by eliminating arbitrary constants a and b from
$(x-a)^2 + (y-b)^2 + z^2 = 1$

6. Find the PDE of all planes having equal intercepts on the x and y axis.

7. Eliminate the arbitrary function f from $z = f\left(\frac{xy}{z}\right)$ **and form the PDE.**

8. Form partial differential equation by eliminating arbitrary function $z = f(xy)$.

9. Form the PDE by eliminating the arbitrary function from $\phi\left[z^2 - xy, \frac{x}{z}\right] = 0$

10. Find the complete integral of $p + q = pq$ **where** $p = \frac{\partial z}{\partial x}$ **and** $q = \frac{\partial z}{\partial y}$.

11. Write down the complete solution of $px + qy + c\sqrt{1+p^2+q^2}$.

12 Find the singular integral of the partial differential equation $z = px + qy + p^2 - q^2$.

13. Find the solution of $px^2 + qy^2 = z^2$

14.Find the general solution of $5\frac{\partial^2 z}{\partial x^2} - 12\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = 0$.

15. Find the general solution of $\frac{\partial^2 z}{\partial x^2} - 7\frac{\partial^2 z}{\partial x \partial y} + 6\frac{\partial^2 z}{\partial y^2} = 0$

16. Solve $(D^2 - DD' + D' - 1)z = 0$

17. Solve $(D^3 - 3DD'^2 + 2D'^3)z = 0$

18. Solve $(D^3 + D^2D' - DD'^2 - D'^3)\ z = 0$.

19. Find the particular integral of $(D^2 - 2DD'^2 + D'^2)z = e^{x-y}$.

20. Find the particular integral of $(D^3 - 3D^2D' - 4DD'^2 + 12D'^3)z = \sin(x + 2y)$.

PART-B

1. Form the PDE by eliminating the arbitrary functions $\varphi(x^2 + y^2 + z^2, ax + by + cz)$.

2. Form the PDE by eliminating the arbitrary functions f and g in $z = xf(x + ct) + g(2x + y)$.

3. Form the partial differential equation by eliminating f and ϕ from $z = f(y) + \phi(x + y + z)$.

4. Form the PDE by eliminating arbitrary function from the relation $Z = y^2 + 2f(\frac{1}{x} + \log y)$

5. Form the PDE by eliminating arbitrary function f and g from $z = x^2 f(y) + y^2 g(x)$

6. Solve $z = px + qy + \sqrt{1+p^2+q^2}$.

7. Find the singular integral of the partial differential equation $z = px + qy + p^2 + pq + q^2$

8. Solve $z = 1 + p^2 + q^2$.

9. Solve $p(1 + q) = qz$

11. Solve $x^2p^2 + y^2q^2 = z^2$

12. Solve $x(y - z)p + y(z - x)q = z(x - y)$.

UNIT - II FOURIER SERIES

PART-A

1. Find the constant term in the Fourier series corresponding to f(x) = $\cos^2 x$ expressed in the interval $(-\pi, \pi)$

2.To which value, the half range sine series corresponding to f(x) = x^2 expressed in the interval (0,2) converges at x=2.

3. If f(x) = $x^2 + x$ is expressed as a Fourier series in the interval (-2, 2) to which value this series converges at x=2.

4. If the Fourier series corresponding to f(x) = x in the interval (0, 2π) is $\frac{a_0}{2} + \sum_1^\infty (a_n \cos nx + b_n \sin nx)$, without finding a_0, a_n, b_n find the value of $\frac{a_0^2}{2} + \sum_1^\infty (a_n^2 + b_n^2)$.

5. State Dirichlet's for a given function to expand in Fourier series.

6. If the Fourier series of the function f(x) = x + x^2 in the interval $-\pi < x < \pi$ is $\frac{\pi^2}{3} + 4\sum_{n=1}^{\infty} (-1)^n \left[\frac{4}{n^2}\cos nx - \frac{2}{n}\sin nx\right]$

then find the value of $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots\dots\dots$

7. Find a Fourier sine series for the function f(x) = 1 , i) 0<x<π ii) (0,2)

8. **If the Fourier series for the function f(x) =** $\begin{cases} 0 & 0 < x < \pi \\ \sin x & \pi < x < 2\pi \end{cases}$ **is**

f (x)= $-\frac{1}{\pi}+\frac{2}{\pi}\left[\frac{\cos 2x}{1.3}+\frac{\cos 4x}{3.5}+\frac{\cos 6x}{5.7}+....\right]+\frac{\sin x}{2}$ **deduce** $\frac{1}{1.3}-\frac{1}{3.5}+\frac{1}{5.7}-.........\infty=\frac{\pi-2}{4}$

9. **What is the constant term** a_0 **and the coefficient of cosnx,** a_n **in the Fourier series expansion of f(x) = x - x^3 in** $(-\pi,\pi)$.

10. **State Parseval's identity for full range expansion of f(x) as Fourier series in** $(0,2l)$

11. **What do you mean by Harmonic analysis.**

12. **In the Fourier expansion of f(x)=** $\begin{cases} 1+\frac{2x}{\pi}, -\pi < x < 0 \\ 1-\frac{2x}{\pi}, 0 < x < \pi \end{cases}$ **in** $(-\pi,\pi)$, **find** b_n.

13. **Find** b_n **in the expansion of** x^2 **as a Fourier series in** $(-\pi,\pi)$

14. **If f(x) is an odd function defined in** $(-l,l)$, **what are the values of** a_0 **and** a_n

15. **State Parseval's Theorem on Fourier series**

16. **Does f(x) = tanx possess a Fourier expansion**

17. **Find** a_n **in expanding** e^{-x} **as Fourier series in** $(-\pi,\pi)$

18. **Find the Fourier constants** b_n **for xsinx in** $(-\pi,\pi)$

19. **State Parseval's identity for the half-range cosine expansion of f(x) in (0, 1).**

20. **Find the root mean square value of the function f(x) = x in (0, *l*).**

Part-B

1. **Find the Fourier series of the function f(x) =** $(\pi-x)^2$, **in** $(0,2\pi)$ **with periodicity** 2π

2. **Find the Fourier series of the function f(x) =**$x(2\pi-x)^2$, **in** $(0,2\pi)$ **with periodicity** 2π **.Also deduce the sum of the series** $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+..........\infty=\frac{\pi^2}{6}$

3. **Obtain the Fourier series for f(x) = 1 + x +** x^2 **in** $(-\pi,\pi)$.

4. **Find the Fourier series for f(x)=**x^2 **in** $(-\pi,\pi)$. **Hence find**

i) $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+..........$ **ii)** $\frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{3^2}-\cdots$ **iii)** $\frac{1}{1^4}+\frac{1}{2^4}+\frac{1}{3^4}+\frac{1}{4^4}........\infty$

5. **Find the Fourier series for f(x) =** $|\cos x|$ **in the interval** $(-\pi,\pi)$

6. **Determine the Fourier expansion of f(x)=x in the interval** $-\pi < x < \pi$

7. **Obtain the Fourier series for the function f(x) =** $\begin{cases} \pi x & 0 \le x \le 1 \\ \pi(2-x) & 1 \le x \le 2 \end{cases}$

8. **. Find the Fourier series for the function f(x) =** $\begin{cases} x & \text{in } 0 < x < 1 \\ 1-x & \text{in } 1 < x < 2 \end{cases}$

Deduce that $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+.......=\frac{\pi^2}{8}$.

9. **Find the Fourier series expansion of the periodic function f(x) of period** $2l$ **define by**

f(x) = $l+x \qquad -l \le x \le 0$

$=l-x \qquad 0 \le x \le l$ **Deduce that** $\sum_{1}^{\infty}\frac{1}{(2n-1)^2}=\frac{\pi^2}{8}$

10. Find the Fourier series of period 2π for the function f(x)= $\begin{cases} 0 & \text{in } (0,\pi) \\ \sin x & \text{in } (\pi,2\pi) \end{cases}$

and hence find the sum of the series $\frac{\mathbf{1}}{\mathbf{1.3}}+\frac{\mathbf{1}}{\mathbf{3.5}}+\frac{\mathbf{1}}{\mathbf{5.7}}+\cdots$

11. Find the Fourier series expansion of $\boldsymbol{f}(\boldsymbol{x})=\begin{cases} \mathbf{1}\ \boldsymbol{for}\ \mathbf{0}<x<\pi \\ \mathbf{2}\ \boldsymbol{for}\ \boldsymbol{\pi}<x<2\boldsymbol{\pi} \end{cases}$

12.Find the Fourier series expansion of $\boldsymbol{f}(\boldsymbol{x})=\begin{cases} -\boldsymbol{x}+\mathbf{1}\ \boldsymbol{for}-\boldsymbol{\pi}<x<0 \\ \boldsymbol{x}+\mathbf{1}\ \boldsymbol{for}\ \mathbf{0}<x<\pi \end{cases}$

UNIT –III (APPLICATION OF PARTIAL DIFFERENTAIL EQUATIONS)

PART – A

1. **In the wave equation $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$ what does c^2stand for?**

2.In the diffusion equation $\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$,what does α^2 stand for

3. **What are all the solutions of one dimensional wave equation?**
4. **What is the basic difference between the solution of one dimensional wave equation and one dimensional heat equation?**
5. **Define steady state condition on heat flow**
6. **What are all the solutions of one dimensional heat equation?**
7. **In steady state conditions derive the solution of one dimensional heat equation.**
8. **State the assumptions in deriving the one dimensional equation(unsteady state)**
9. **A rod 30 cm long has its ends A and B kept at 20^0 C and 80^0 C respectively until steady state conditions prevail. Find the steady state temperature in the rod.**
10. **Write down the two dimensional heat flow equation in steady state.**
11. **Write down the possible solutions of two dimensional heat equationsin steady state.**
12. **Write any two solutions of Laplace equation $u_{xx} + u_{yy} = 0$ involving exponential terms in $x \text{ and } y$.**
13. **Write down the boundary and initial conditions for the transverse vibrations of the string of length l with fixed ends with initial displacement $y = f(x)$.**
14. **If the string of length l are fixed and the mid point of the string is drawn aside through a height h and the string is released from rest, state the initial and boundary conditions.**
15. **A tightly stretched string of length $2l$ has its ends fastened at x = 0 and x = $2l$. The midpoint of the string is then taken to a height b and then released from rest in that position. Write the initial and boundary conditions.**
16. **A tightly stretched string with end points $x = 0 \text{ and } x = l$ is initially at rest in equilibrium position.If it is set vibrating giving each point velocity $\lambda x(l - x)$. Write the initial and boundary conditions.**
17. **The ends A and B of a rod of length 10 cm long have their temperature kept 20^0 C and 70^0 C.find the steady state temperature on the rod.**
18. **An infinitely long plane uniform plate is bounded by two parallel edges and an end at right angles to them. The breadth is π, this edge $u(x, 0)$ is maintained at a temperature 60^0C at all points & the other edges are at zero temperature. Write down the boundary conditions for finding the steady-state temperature.**
19. **State any two laws which are assumed to derive one dimensional heat equation.**
20. **Find the steady state temperature distribution in a rod of length l cm whose ends x = 0 and x = $l$are kept at 0^0C and80^0 C.**

PART –B

1. A tightly stretched string of length l has its end fastened at x = 0, x = l. At $t = 0$,the string is in the form $y(x,0) = kx(l - x)$ and then released .Find the displacement of any point on the string at a distance of x from one end at time t>0.

2.A tightly stretched string of length l has its ends fastened at x = 0 and x = l. The midpoint of the string is then taken to a height b and then released from rest in that position. Obtain an expression for the displacement of the string at any subsequent time.

3. A tightly stretched string of length $2l$ has its ends fastened at x = 0 and x = $2l$. The midpoint of the string is then taken to a height b and then released from rest in that position. Find the lateral displacement of a point of the string at time t from the instant of release.

4. A slightly stretched string of length l has its ends fastened at x = 0 and x = l is initially in a position given by $y(x,0) = y_0 sin^3 \frac{\pi x}{l}$.If it is released from rest from this position,find the displacement $y$at any distance $x$from one end and at any time.

5. If a string of length l is initially at rest in its equilibrium position and each of its points is given the velocity $v_0 \sin^3 \frac{\pi x}{l}$, determine the displacement of a point distant x from one end at time t.

6. A sting of length l is initially at rest in its equilibrium position and motion is started by giving each of its points a velocity given by $v = \begin{cases} cx & if\, 0 \le x \le \frac{l}{2} \\ c(l-x) & if\, \frac{l}{2} \le x \le l \end{cases}$.

Find the displacement function y(x, t).

7. A tightly stretched string with end points $x = 0 \; and \; x = l$ is initially at rest in equilibrium position.If it is set vibrating giving each point velocity $\lambda x(l - x)$, then show that

$$y(x,t) = \frac{8\lambda l^3}{a\pi^4} \sum_{n=1,2,3,\dots} \frac{1}{n^4} \sin\frac{n\pi x}{l} . sin\frac{n\pi at}{l}.$$

8. A rod 30 cm long has its ends A and B kept at 20^0 and 80^0 respectively until steady state conditions prevail. The temperature at each end is then suddenly reduced to 0^0C and kept so. Find the resulting temperature function u(x, t) taking x = 0 at A.

9. The ends A and B of a rod l cm long have the temperature40^0 and 90^0 respectively until steady state conditions prevail. The temperature at A is suddenly raised to 90^0 and at the same time that at B is lowered to 40^0. Find the temperature distribution in the rod at time t.

10. Find the solution of the equation$\frac{\partial u}{\partial t} = a^2 \frac{\partial^2 u}{\partial x^2}$that satisfies the conditions $u(0,t) = 0 \,, u(l,t) = 0 \,, for\; t > 0, and\; u(x,0) = \begin{cases} x, 0 \le x \le l/2 \\ l - x, l/2 < x < l \end{cases}$

10. A square plate is bounded by the lines x = 0, y = 0, x = 20, y = 20. Its faces are insulated. The temperature along the upper horizontal edge is given by u(x, 0) = x (20-x) when 0<x<20 while the other three edges are kept at 0^0 C. Find the steady state temperature in the plate.

11. A rectangular plate with insulated surface is 10cm wide and so long compared to its width that it may be considered infinite in length without introducing appreciable error. The temperature at short edge y = 0 is given by $u = \begin{cases} 20x & , \quad 0 \le x \le 5 \\ 20(10 - x), & 5 \le x \le 10 \end{cases}$. And all the other three edges are kept at 0^0 C. Find the steady state temperature at any point in the plate.

12. A rectangular plate with insulated surface is 10cm wide. The two long edges and one short edge are kept at0^0 C,while the temperature at short edge x = 0 is given by

$$u = \begin{cases} 20y, & 0 \le y \le 5 \\ 20(10-y), & 5 \le y \le 10 \end{cases}$$ **Find the steady state temperature at any point in the plate.**

UNIT- IV (FOURIER TRANSFORMS)

PART-A

1. State the Fourier integral theorem.

2. If $F_C(s)$ is the Fourier Cosine transform of $f(x)$, prove that the Fourier cosine transform of $f(ax)$ is $\frac{1}{a} F_c\left(\frac{s}{a}\right)$

3. If $F(s)$ is the Fourier transform of $f(x)$, write the formula for the Fourier of $f(x)cos(ax)$ in term of F.

4. State the convolution theorem for Fourier transforms.

5. If F(s) is the Fourier transform of f(x), find the Fourier transform of $f(x-a)$.

6. If $F_s(s)$ is the Fourier sine transform of f(x), show that $F_s\{f(x) \cos ax\} = \frac{1}{2}[F_s(s+a)+F_s(s-a)]$

7. Find the Fourier transform of f(x) if $f(x) = \begin{cases} 1, & |x| < a \\ 0, & |x| > a > 0 \end{cases}$

8. Find Fourier Cosine transform of e^{-x}

9. Find: $F\{x^n f(x)\}$

10. F $\left\{\frac{d^n f(x)}{dx^n}\right\}$ **in terms of F.Tof f(x)**

11. Write the Fourier transform pair.

12. Find Fourier sine transform of $\frac{1}{x}$.

13. State the Fourier transforms of the derivatives of a function.

14. Find the Fourier transform of $e^{-\alpha|x|}$ **,** $\alpha > 0$

15.Find the function f(x) whose sine transform is $\frac{e^{-as}}{s}$.

16. If F {f(x)} = $\overline{f}(s)$ **then give the value of F {f (ax)}**

17.Find Fourier transform of $f(x) = \begin{cases} 1 & |x| \le 1 \\ 0 & |x| > 1 \end{cases}$

18. Find the Fourier sine transform of $f(x)=e^{-x}$

19. Prove that $F[f(ax)] = \frac{1}{a} F\left(\frac{s}{a}\right)$, a >0

20. State Modulation theorem on Fourier transforms.

PART-B

1. Find the Fourier transform of $f(x)=\begin{cases}1 & \text{for } |x|<1\\ 0 & \text{otherwise}\end{cases}$**, Hence prove that** $\int_0^\infty \frac{\sin x}{x}dx=\int_0^\infty \frac{\sin^2 x}{x^2}dx=\frac{\pi}{2}$

2.Find the Fourier transform of $f(x)=\begin{cases}1-|x| & \text{if } |x|<1\\ 0 & \text{if } |x|\geq 1\end{cases}$ **Hence deduce** $\int_0^\infty\left(\frac{\sin t}{t}\right)^4 dt=\frac{\pi}{3}$

3.Show that the Fourier transform of f(x)=$\begin{cases}a^2-x^2, & |x|<a\\ 0 & |x|>a\end{cases}$

is $2\sqrt{\frac{2}{\pi}}\left(\frac{\sin\lambda a-\lambda a\cos\lambda a}{\lambda^3}\right)$**. Hence deduce that** $\int_0^\infty \frac{\sin t-t\cos t}{t^3}dt=\frac{\pi}{4}$**.**

4.Find the Fourier Transform of $e^{-a|x|}, a>0$**. Hence deduce that** $F\left[xe^{-a|x|}\right]=i\sqrt{\frac{2}{\pi}}\frac{2as}{\left(a^2+s^2\right)^2}$**.**

5.Find the Fourier transform of $e^{-a|x|}$ if $a>0$. **Deduce that** $\int_0^\infty \frac{1}{(x^2+a^2)^2}dx=\frac{\pi}{4a^3}$ if $a>0$

6.Find the Fourier transform of the function f(x) defined by

f(x) =$\begin{cases}1-x^2 & |x|\leq 1\\ 0 & |x|>1\end{cases}$ **Hence prove that (i)** $\int_0^\infty\left\{\frac{\sin s-s\cos s}{s^3}\right\}\cos\frac{s}{2}\,ds=\frac{3\pi}{16}$

(ii) $\int_0^\infty\left(\frac{\sin s-s\cos s}{s^3}\right)^2 ds=\frac{\pi}{15}$

7.Find the Fourier Transform of f(x) given by

f(x) = $\begin{cases}a-|x|, & if\ |x|<a\\ 0, & if\ |x|>a>0\end{cases}$ **Hence deduce that** $\int_0^\infty\left(\frac{\sin t}{t}\right)^2 dt=\frac{\pi}{2}$**.**

8.If $f(x)=xe^{-\frac{x^2}{2}}$ **is self - reciprocal under the FourierCosine Transform then prove that** $g(x)=xe^{-\frac{x^2}{2}}$ **is self - reciprocal under the Fourier Sine Transform.**

9. Find the Fourier sine transform of $f(x)=\begin{cases}\sin x, & 0<x<\pi\\ 0 & \pi\leq x<\infty\end{cases}$

10. Find the Fourier cosine transform of e^{-4x}**.**

Deduce that $\int_0^\infty \frac{\cos 2x}{x^2+16}dx=\frac{\pi}{8}e^{-8}$ **and** $\int_0^\infty \frac{x\sin 2x}{x^2+16}dx=\frac{\pi}{2}e^{-8}$

11. State and prove convolution theorem for Fourier transforms.

12. Find the Fourier sine transform of $xe^{-\frac{x^2}{2}}$

UNIT – V: Z – TRANSFOMS AND DIFFERENCE EQUATIONS

PART – A

1. **Define Z – Transform of the sequence** $\{f(n)\}$.
2. **Find the Z – Transform of unit step sequence.**
3. **Find** $Z\left(3^{n+2}\right)$
4. **Find** $Z\left[\cos^2 \frac{n\pi}{2}\right]$.
5. **Find** $Z[n+2]$.
6. **Find** $Z\left[\frac{1}{n}\right]$.
7. **Find** $Z\left[\frac{a^n}{n!}\right]$.
8. **Find** $Z[a^n n]$.
9. **Prove that** $Z[a^n] = \frac{z}{z-a}$.
10. **Find** $Z\left[\frac{1}{n(n+1)}\right]$
11. **Find** $Z[n^2]$.
12. **Find** $Z\left[\frac{1}{(n+1)!}\right]$.
13. **State the Initial and Final value theorem.**
14. **Prove that** $Z[f(n+1)] = zF(z) - zf(0)$.
15. **Find the Z transform of** $n(-1)^n$.
16. **Find the inverse Z- transform of** $\frac{z}{(z-1)(z-2)}$.
17. **Form a difference equation by eliminating the arbitrary constants from** $y_n = A + B.2^n$.
18. **Form a difference equation by eliminating the arbitrary constants from** $U_n = a2^{n+1}$.
19. **Solve** $y_{n+1} - 2y_n = 0$ **given that** $y(0)=2$.
20. **State Convolution theorem in Z – Transforms.**

PART – B

1. **Find** $(i) Z\left[r^n \cos n\theta\right]$, $(ii)\ Z\left[r^n \sin n\theta\right]$ $iii) Z(e^{-at} \cos bt)$
2. **Find the Z-transform of** $\sin^3\left(\frac{n\pi}{6}\right)$**.**
3. **Find** $(i)\ Z\left[\sin \frac{n\pi}{2}\right]$, $(ii)\ Z\left[2^n \sin \frac{n\pi}{2}\right]$ **.**
4. **Find the Z – Transform of** $f(n)=\frac{2n+3}{(n+1)(n+2)}$ **.**
5. **Find the z-transform of** $\sinh at \sin bt$**.**
6. **Find the Z transform of** $\frac{1}{(n+1)(n+2)}$ **and** $2^n \cos\frac{n\pi}{2}$**.**
7. **If** $Z\{f(n)\} = F(z)$**, prove that** $Z[nf(n)] = -z\frac{d}{dz}F(z)$**.**
8. **Find the inverse Z – Transform of** $\frac{z(z^2 - z + 2)}{(z+1)(z-1)^2}$.
9. **Find the inverse Z -Transform of** $\frac{z^3+3z}{(z-1)^2(z^2+1)}$.
10. **Find the inverse Z -Transform of** $\frac{z^2}{(z+2)(z^2+4)}$ **by the method of Partial fraction.**
11. **Find** $Z^{-1}\left(\frac{z}{z^2+7z+10}\right)$ **by convolution theorem.**
12. **. Find i)** $Z^{-1}\left[\frac{z^2}{(z-1)(z-3)}\right]$ **ii)** $Z^{-1}\left[\frac{z^2}{(z-a)(z-b)}\right]$ **iii)** $\frac{12z^2}{(3z-1)(4z+1)}$ **usingconvolution theorem.**